**UOG-UESTC Joint School of**

**University of Electronic Science and Technology of China**

**Signals and Systems —** **Fall 2015**

**Final Exam**

**9:00—11:00 AM, Saturday, January 26, 2016**

-----------------------------------------------------------------------------------------------------------------------------------------------------------

**Notice: Please make sure that both your UESTC and UoG Student IDs are written on the top of every sheet.This examination is closed-book and the use of a calculator or a cell phone is not permitted. All scratch paper must be adequately labeled.**  **Unless indicated otherwise, answers must be derived or explained clearly. Please write within the space given below on the answer sheets.**

**All questions are** **compulsory. There are 6 questions and a maximum of 100 marks in total.**

**The following table is for grader only:**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Question** | **1** | **2** | **3** | **4** | **5** | **6** | **Total** | **Grader** |
| **Score** |  |  |  |  |  |  |  |  |

-----------------------------------------------------------------------------------------------------------------------------------------------------------

**Question1 (5x2=10 points)**

**Suppose is the frequency response of an ideal continuous-time low pass filter (LPF), and its cutoff frequency is .**



**(1) Find and sketch the unit impulse response of this LPF.**



**(2) Is it possible to realize such an idea filter in practice? Justify your answers.**

**Solution:**

**(1)**

**For an ideal LPF , its unit impulse response is .**

****

**(2) The unit impulse response of this ideal LPF is a noncausal signal crossing the whole time range, which is impossible to realize the corresponding noncausal physical system. In addition, the constant value of the passband and stopband of the ideal filter produces the unlimited ripples of , which also makes it hard to approach.**



ScoreScore

**Question2 (4x5=20 points)**

**A none distortion linear system is highly desirable in many real-world applications, e.g., in video conferencing and communication systems. The transfer function of such none distortion linear system comprises only a perfect gain constant *K* and a perfect delay , which builds the relationship between the input signal  and the output  as .**

**(a) Determine the frequency response  of the none distortion linear transmission system.**

**(b) Sketch the magnitude spectrum and phase spectrum of , respectively.**

**(c) With the help of the plots in (b), explain how the none distortion linear system will change each frequency component of the input signal.**

**(d) Assume there are two frequency components in , 500Hz and 1000Hz, which will be delayed 5 second and 7 second after a transmission system without amplitude attenuation, respectively. Is this system a non distortion linear system? Justify your answer.**

**Solution:**

**(a) Because **

**So, the unit impulse response , then .**

**(b) The magnitude spectrum and phase spectrum of  are:**

 

**(c) The none distortion linear system will pass all the frequency components of the input signal with a constant gain *K*; and give each frequency components a phase shift at the frequency  is a linear function , which means a simply time shift of the input signal.**

**(d) No. Because for the frequency components 500Hz and 1000H, they have different time shift in time domain, which means phase shift is a nonlinear function of  and violate the rules of the non distortion linear system.**

Score

**Question 3 (20 points)**

**Sampling is a critical step in obtaining discrete-time signal from continuous-time signal. Under certain conditions, a continuous-time signal can be perfectly reconstructed by the samples at points equally spaced in time, which is known as *Sampling Theorem*.**

**(a) Describe the *Sampling Theorem*.**

**(b) The frequency range for a person’s speech is mostly within 300Hz to 3400Hz. Assuming the maximum frequency is 4000Hz, what is the requirement for the sampling frequency according to the *Sampling Theorem*?**

**(C) The *Sampling Theorem*** **is** **obtained by impulse-train sampling. However, the impulse-train is an ideal signal. Instead a periodic pulse-train signal with sampling period *T* is used as a sampling function, as shown in Figure 1, where  is the pulse width.**



Figure 1

**In such a case, is the *Sampling Theorem* still valid? Suppose  is a band-limited signal with maximum frequency . The sampling function is , and the sampled signal is . Find their relationships in time domain and frequency domain to justify your answer.**

**Solution:**

***(a) Sampling Theorem*: Let be a band-limited signal with  for . Then  is uniquely determined by its samples , if , Where .**

**(b) According to the *Sampling Theorem*, the sampling frequency .**

**(c) Yes, the *Sampling Theorem* is still valid.**

**In time domain **

**In frequency domain **

**And 🡪**

****

**So **

**The above spectrum show that the sampling process with the periodic pulse-train signal also makes a periodical copies of the original spectrum of , and the difference with the ideal impulse-train sampling is that the envelope of the magnitude spectrum of the sampled signal becomes a sinc function. If ,  can still be uniquely determined by its samples.**

**So, the *Sampling Theorem* is still valid.**

Score

**Question 4 ( 20 points)**

**A periodic signal  is applied to a RC circuit as in Figure 2. Suppose the frequency KHz, and the amplitude of the source voltage V,** **, .**

**(a) Determine the frequency response  of the circuit.**

**(b) Find the Fourier series representation of the input signal .**

**(c) Determine the Fourier series coefficients of the output signal .**



Figure 2.

**Solution:**

**(a) **

**For , , so **

**(b) **

****

**For  KHz, V**

**So, ,  ( *k* is even, =0)**

**(c) As to the output signal **

****

****

Score

**Question 5 ( 15 points)**

**Consider an LTI system with the unit impulse response , where is an unknown constant. When the input to the system is , the output is .**

**(a) Determine the system function  and sketch the pole-zero plot indicating the region of convergence (ROC) of . ( 7 points )**

**(b) Determine causality and stability of this system. ( 2 points )**

**(c) Draw a direct-form block diagram representation of this system. ( 6 points )**

**Solution:**

**(a) **

**, , **

Then we can get .

**So,  **

****

**(b) The system is causal and stable.**

**(c) **

****

Score

**Question 6 (15 points)**

**Consider an LTI system with input  and its corresponding output .**

**(a) Show the system function . ( 5 points )**

**(b) Determine the unit impulse response . Discuss causality and stability of this system. ( 6 points )**

**(c) Find the output of this system, when the input signal is . ( 4 points )**

**Solution:**

**(a) , **

**, **

** , **

**(b) **

****

**The system is stable, not causal.**

**(c)  is the eigenfuction of the system**

****